Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Differential and Integral Calculus

Subject Code: 4SC04DIC1 **Branch: B.Sc. (Physics)**

Semester: 4 **Date: 26/04/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Evaluate: $\int_1^2 \int_0^1 xy\ dx\, dy$ **(02)**

b) Evaluate: $\int_0^3 \int_0^2 \int_0^1 dx\, dy\, dz$. **(02)**

c) A particle moves along the $x = t^3 + 1, y = t^2, z = 2t + 5$, where t is the time. Find the component of its velocity at time $t = 1$. **(02)**

d) Prove that $curl\ (grad\ \phi) = \bar{0}$ where ϕ is scaler valued function. **(02)**

e) When a vector function $\bar{F}$ is irrotational? **(01)**

f) State Green's Theorem. **(01)**

g) State Stoke's Theorem. **(01)**

h) Write a formula of curvature in polar form. **(01)**

i) Define: Node. **(01)**

j) What are the conditions to check the curve $f(x, y) = 0$ having a double point as cusp? **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the directional derivatives of $\phi = 2xy^2 + yz^2$ at the point $(2, -1,1)$ in the direction of the vector $i + 2j + 2k$. **(05)**

b) Find divergence and curl of $\bar{v} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at $(2, -1,1)$. **(05)**

c) Find value of m if $\bar{F} = (x + 2y)i + (my + 4z)j + (5z + 6x)k$ is solenoidal. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Evaluate $\int_c \bar{F}\ d\bar{r}$ where $\bar{F} = (x^2 + y^2)i - 2xy\, j$ and c is rectangle in the $xy -$ plane bounded by $y = 0, x = a, y = b, x = 0$. **(07)**

b) Find work done in moving a particle from $A(1,0,1)$ to $B(2,1,2)$ along the straight line AB in the force field $\bar{F} = x^2 i + (x - y)j + (y + z)k$. **(07)**

Q-4 **Attempt all questions** **(14)**

a) Using polar coordinates, find $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\ dx\, dy$ **(05)**

b) Evaluate: $\int_1^3 \int_{\frac{1}{x}}^1 \int_0^{\sqrt{xy}} xyz\ dx\, dy\, dz$. **(05)**

	c)	Evaluate: $\iint_R x\sqrt{1-x^2}\,dx\,dy$, where $R: 0 \le x \le 1, 2 \le y \le 3$.	**(04)**
Q-5		**Attempt all questions**	**(14)**
	a)	Evaluate $\iint_R y\,dx\,dy$ where R is region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.	**(05)**
	b)	Evaluate $\iint_R \sqrt{x+y}\,\,dx\,dy$, where R is the parallelogram bounded by the lines $x+y=0, x+y=1, 2x-3y=0, 2x-3y=4$.	**(05)**
	c)	Change the order of integration $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	Verify Green's theorem for $\int_c[(x-y)dx + 3xydy]$, where c is the boundary of the region bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$.	**(09)**
	b)	Using Stoke's theorem for the vector field $\bar{F} = (x+y)i + (y+z)j - xk$ and S is the surface of the plane $2x+y+z=2$ which is in the first octant.	**(05)**
Q-7		**Attempt all questions**	**(14)**
	a)	Solve: $p\tan x + q\tan y = \tan z$.	**(05)**
	b)	Solve: $(mz-ny)p + (nx-lz)q = ly - mx$.	**(05)**
	c)	Form a partial differential equation by eliminating arbitrary constants from the equation $z = a(x+y) + b$ where a, b are constants.	**(04)**
Q-8		**Attempt all questions**	**(14)**
	a)	Prove that radius of curvature for the curve $y = f(x)$ is $\frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$.	**(05)**
	b)	Find radius of curvature for the curve $r = a(1-cos\theta)$.	**(05)**
	c)	Find the double points of the curve $x^3 + y^3 - 12x - 27y + 70 = 0$	**(04)**